Hence the determinant of the matrix is given by

$$|M| = a_{00}a_{11}a_{22}a_{33}a_{44}a_{55}B_{15}B_{25}B_{35}B_{45}B_{55}. \tag{67}$$

In order to avoid indeterminant forms, it is necessary that none of the main diagonal terms of the original matrix be zero.

This derivation may be easily extended to any order matrix to give

$$|M| = a_{00}a_{11}a_{22} \cdots a_{nn}B_{1n}B_{2n} \cdots B_{nn} \tag{68}$$

and hence the given theorem is proved.

ACKNOWLEDGMENT

The author wishes to thank B. B. Bossard of RCA, who generated the interest in this investigation, and Dr. G. Weiss of the Polytechnic Institute of Brooklyn, who supported and acted as advisor for this project. The author is especially indebted to E. W. Sard of Airborne Instruments Laboratory for his careful review of this manuscript, and for suggesting derivations which made possible the general noise figure results given in the subsection, *Minimum Noise Conditions for Low $\omega_s/m_1\omega_c$.*

A Simple Diode Parametric Amplifier Design for Use at S, C, and X Band

C. S. AITCHISON, R. DAVIES, AND P. J. GIBSON

Abstract—**A simple diode parametric amplifier is described which has been designed for use at *S*, *C*, and *X* band frequencies. Bandwidth and noise measurements show the performance to be substantially in agreement with the theoretical predictions. Details are given of compensating circuits which improve the gain-frequency response of the amplifier by up to 5.6 times.**

I. Introduction

MANY PARAMETRIC amplifier designs involve the use of distributed signal and idling circuits with consequent reduction of the gain bandwidth performance which is available from the amplifier. The techniques described in this paper show that lumped techniques can be utilized to give improved bandwidths, and that reactance compensating networks can be used to increase the gain-frequency response even further.

A. General Principles of Parametric Amplifier Circuit Design

A satisfactory diode parametric amplifier design must fulfill the following requirements:

a) The diode must be coupled to a circuit which provides the appropriate resonating reactance and appropriately overcoupled source resistance at the signal frequency. The diode must also be coupled to a reactance (preferably lossless) which provides a conjugate reactance at a convenient idling frequency. The noise figure F of a negative-resistance amplifier can be shown [1] to be given by the expression

$$F = 1 + \frac{R_d}{R_g} + A\frac{\omega_1}{\omega_2}\frac{(R_d + R_g)}{R_g} \tag{1}$$

where R_d is the diode spreading resistance, R_g the source resistance, A the ratio of negative resistance in the signal circuit to total positive resistance in the signal circuit, ω_1 the signal frequency, and ω_2 the idling frequency. R_g is selected so that the diode current due to pumping is small, typically one microampere, in order that shot noise associated with this current shall remain negligible. There is no contribution to (1) from the circulator since it is assumed to have infinite isolation and zero insertion loss.

b) Signal, idling, and pump energies must be confined to the appropriate regions of the circuit in order that maximum operating bandwidth can be obtained and in order that minimum pump power will be required to operate the amplifier.

c) Both signal and idling circuits should be lumped (small compared with the wavelength) or, if distributed, have an electrical length less than one-tenth of the wavelength in order that the Q of the circuit shall not be increased by the distributed nature of the reactance. The available gain-bandwidth product ($G_{av}^{1/2}B$) of a negative resistance amplifier operated in conjunction with a circulator is given by the

Manuscript received May 16, 1966; revised August 30, 1966. This paper was presented at the 1965 Symposium on Microwave Applications of Semiconductors, London, England.

The authors are with the Systems Division, Mullard Research Laboratories, Redhill, Surrey, England.

expression

$$G_{av}^{1/2}B = 2\left(\frac{1}{B_s} + \frac{1}{B_i}\right)^{-1} \quad (2)$$

where B_s is the unpumped signal circuit bandwidth, and B_i is the unpumped idling circuit bandwidth. It is therefore necessary that the signal and idling bandwidths should not be unnecessarily restricted.

d) The pump circuit should be as simple as possible in order that the pump power which is matched into the structure containing the diode shall be dissipated in the diode and not in the supporting structure.

e) The design should be such that the stray reactances are minimized since, except in special circumstances, these decrease the bandwidth.

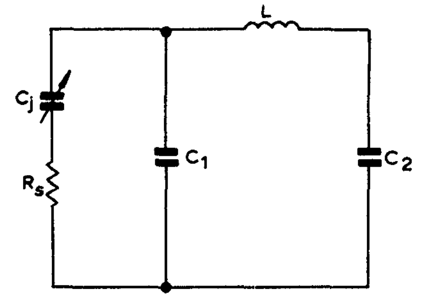
II. Equivalent Circuit of the Varactor Diode

The equivalent circuit of a varactor is shown in Fig. 1 and consists of the spreading resistance R_s in series with the junction capacitance C_j across which is an internal stray capacitance C_1. The lead inductance appears in series with these components and the stray capacitance due to the encapsulation C_2 appears in shunt with the diode. It can be shown from this equivalent circuit that the reactance goes through a zero and a pole as the frequency is increased. Measurement of the VSWR and position of minimum yields a Smith chart impedance plot of the type shown in Fig. 2, which is for the prototype VX3368 diode measured in a 50 ohm coaxial line [2]. This shows that the reactance of the diode is capacitive up to the series-resonant frequency, above which it is inductive until the parallel resonant frequency is reached. Thereafter the impedance is capacitive. A number of diodes have been evaluated and typical results together with the diode figure of merit (γf_c) are shown in Table I. These measurements were made using coaxial standing wave techniques at frequencies up to 18 GHz. Where the parallel resonance occurs at a higher frequency the measurement technique described in Roberts [2] has been applied.

III. Design of Amplifier

The amplifier design uses the diode parallel resonance to support the idling current while the series resonance, suitably modified, supports the signal current.

A schematic drawing of the amplifier is shown in Fig. 3. The amplifier is seen to consist of a coaxial line containing the diode and a waveguide section. The coaxial circuit consists of a 50 ohm input line A followed by a quarter-wave transformer l_1 and filter section l_2, l_3, and l_4. The coaxial lines l_2, l_3, and l_4 are each a quarter-wavelength long at the idling frequency and the corresponding impedances Z_2, Z_3, and Z_4 are chosen so that at the signal frequency the filter is equivalent to a quarter-wavelength of line of the required impedance so as to form the second section of a double



C_j = junction capacitance
R_s = spreading resistance
C_1 = the internal stray capacitance
C_2 = the external stray capacitance
L = lead inductance

Fig. 1. Equivalent circuit of varactor diode.

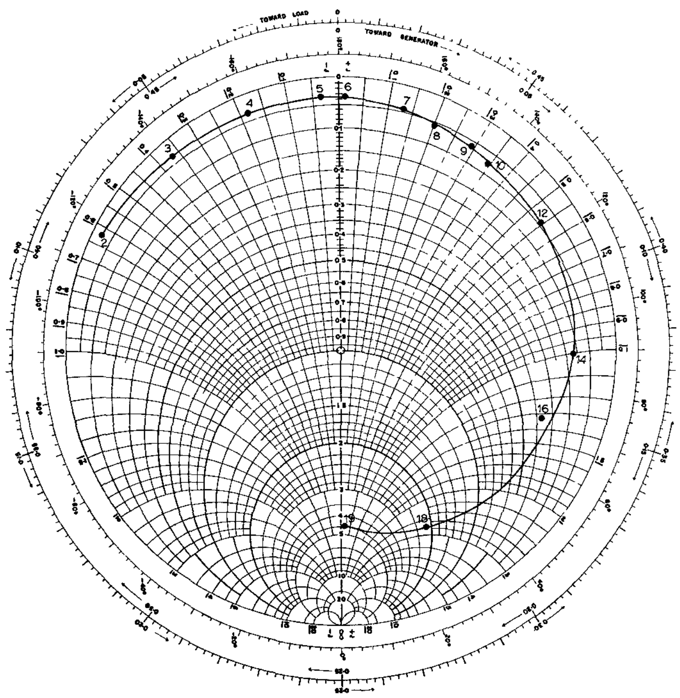


Fig. 2. Impedance on 50 Ω line. Prototype VX3368. Smith chart large ratio 0.0–1.0 VSWR.

TABLE I

Diode Type	Series Resonance GHz	Parallel Resonance GHz	γf_c GHz
VX3368 (Prototype)	5–6	19	—
VX3368	9–10	30	30
MS 264	9–10	—	—
MS2006	9–10	24	24
ZC 25B	6–7	17	18

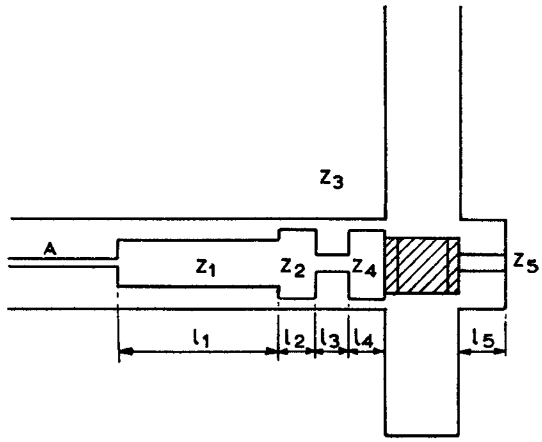


Fig. 3. Schematic diagram of the amplifier. l_1 and l_2, l_3, and l_4 form a double quarterwave transformer at the signal frequency and l_2, l_3, and l_4 are quarter-wave sections at the idling frequency. The diode is shaded. The section l_5 resonates the signal circuit and is a quarter wavelength at the idling frequency.

quarter-wave transformer. Expressions for Z_2, Z_3, and Z_4 (which is equal to Z_2) are described in Appendix I in terms of the signal and idling frequencies and the equivalent impedance.

The diode is placed in series with the double quarter-wave transformer across a waveguide which is cut off at the signal and idling frequencies and which propagates the pump frequency. In series with the diode is a length of coaxial line l_5 which is a quarter-wavelength long at the idling frequency and is essentially a lumped inductance at the signal frequency (since it is short compared with the wavelength). The impedance of l_5, Z_5 is adjusted to make the signal circuit resonate at the required frequency.

It can thus be seen that the idling energy generated in the diode is confined to the region of the diode by the quarter-wavelength section l_5. The unpumped bandwidth of the idling circuit will be determined by the diode equivalent circuit modified by the quarter-wavelength section l_5. Appendix II shows that the Q of the diode alone at the idling frequency is increased by the factor

$$1 + (32\pi^2 C_s^2 Z_0 L f^3)^{-1}$$

by the addition of the short-circuited quarter-wavelength line of impedance Z_0 where C_s is the external stray capacitance of the diode, L is the diode lead inductance, and f the idling frequency. The second term in this factor is usually small compared with the first.

The signal-circuit bandwidth is determined entirely by the diode since the resonating inductance is substantially lumped at the signal frequency. Pump energy is matched into the waveguide containing the diode by means of a suitably placed waveguide short circuit behind the diode and a variable admittance system in front of the diode. The filter section presents a low reactance at the pump frequency as well as the idling frequency, and pump energy does not propagate in the coaxial system.

IV. Operation of the Amplifier

A. Diode Specification

The theoretical noise figure for the amplifier is calculated from (1). The figure of merit for a varactor diode is given by γf_c product, where γ is

$$\frac{C_{\max} - C_{\min}}{2(C_{\max} + C_{\min})}$$

and f_c is the cutoff frequency. $C_{\max}$ and $C_{\min}$ are defined in Appendix III.

The γf_c product of the diode required for an amplifier of signal frequency f_1 and idling frequency f_2 can be shown (Appendix III) to be given by

$$\gamma f_c = \sqrt{f_1 f_2 \left(1 + \frac{R_g}{R_d}\right)}. \tag{3}$$

The ratio R_g/R_d is selected to give the required noise figure at the desired signal frequency and the idling frequency prescribed by the parallel resonance of the diode.

B. Setting-Up Procedure

The signal circuit is resonated by adjustment of Z_5 and the center frequency overcoupling ratio R_g/R_d determined by a VSWR measurement of the amplifier at the signal frequency, taking precautions to avoid perturbing the diode impedance with the measuring power. At the center frequency the VSWR is a minimum and this minimum is the overcoupling ratio used in calculating the noise figure.

The idling frequency is determined initially by the impedance measurement of the diode as previously described and finally by pumping the amplifier at the sum of the signal and predicted idling frequencies. A plot of pump frequency and diode current against signal frequency at constant gain (usually 20 dB) shows a minimum in the diode current and this determines the center frequency and actual idling frequency under operating conditions.

The unpumped signal circuit bandwidth can be determined by means of the VSWR measurement or more conveniently by plotting the signal circuit response using a signal generator, attenuator, and valve voltmeter to monitor the detected diode voltage. The idling circuit unpumped bandwidth can be determined by measurement of the slope of the graph of pump frequency against signal frequency since it can be shown that the idling-to-signal bandwidth ratio is given by the expression

$$B_1 = B_s\left(\frac{\Delta\omega_3}{\Delta\omega_1} - 1\right) \tag{4}$$

as shown in Appendix IV.

Measurement of the unpumped idling and signal bandwidths enables a comparison to be made of the predicted bandwidth using (2) and the observed bandwidth.

V. Amplifier Performance

A. Operation of the Amplifier at S band with the MS2006 Diode

An amplifier of this type has been operated at *S* band using the diode type MS2006. In this case the natural idling resonance was reduced to 16 GHz by the addition of a polytetrafluorethelyne sleeve to the diode. Figure 4 shows the unpumped circuit response determined by means of a signal generator, attenuator and a valve voltmeter and indicates a bandwidth of 470 MHz. Figure 5 is the VSWR plot of the signal circuit showing that the R_g/R_d ratio is 12.5. Figure 6 shows the variation of pump frequency, diode current, and bandwidth as a function of signal frequency at 20 dB gain, demonstrating that the current is at a satisfactorily low value of two microamperes at the center of the tuning range. Noise figure measurements, using a switched circulator and coaxial loads at room and liquid nitrogen temperatures, indicate a noise temperature of 99°K for the amplifier alone. A plot of overall noise figure with variation of local oscillator frequency is shown in Fig. 7, demonstrating an overall noise figure minimum of 1.8 dB with a following receiver noise figure of 10 dB. The variation of overall noise factor with gain is shown in Fig. 8.

The theoretical amplifier noise temperature calculated using (1) is 84°K. Using the slope of Fig. 6, and (4) the unpumped idling bandwidth is calculated as 845 MHz which predicts an operating bandwidth at 20 dB gain of 58 MHz. The typical observed bandwidth is 65 MHz.

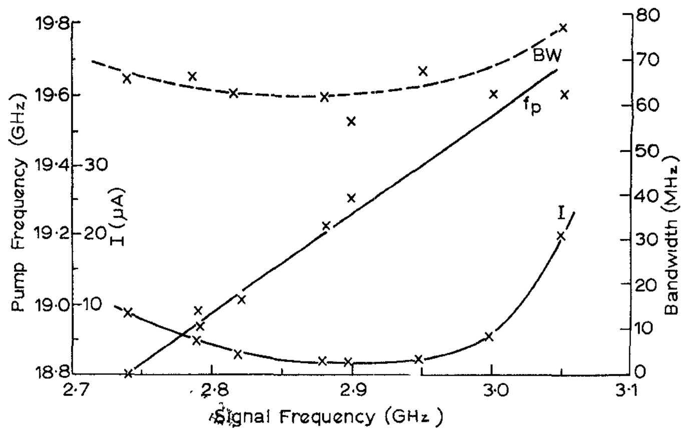


Fig. 6. Pump frequency, diode current and bandwidth at 20 dB gain as a function of signal frequency.

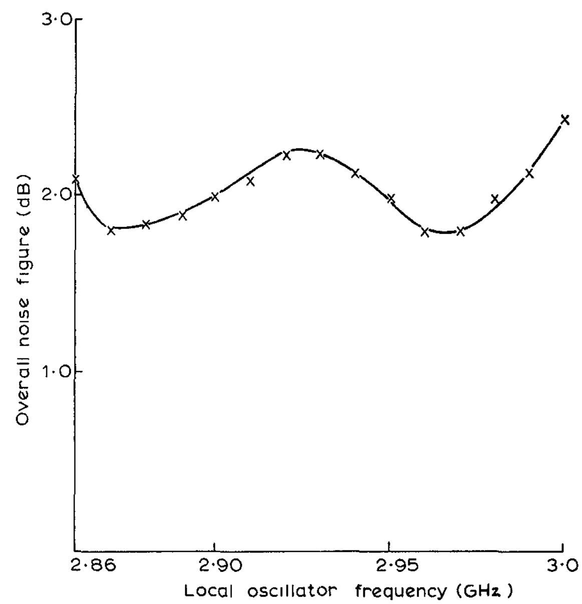


Fig. 7. Variation of overall noise figure with local oscillator frequency.

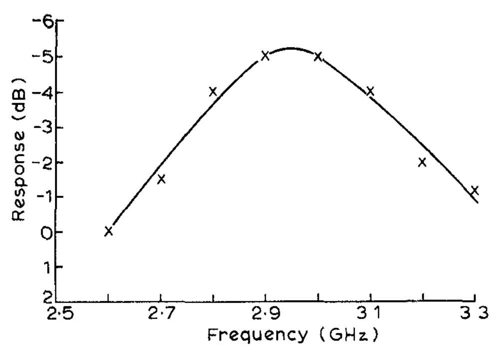


Fig. 4. Signal circuit response.

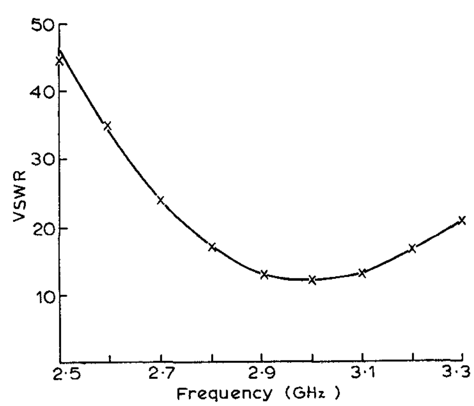


Fig. 5. VSWR of signal circuit.

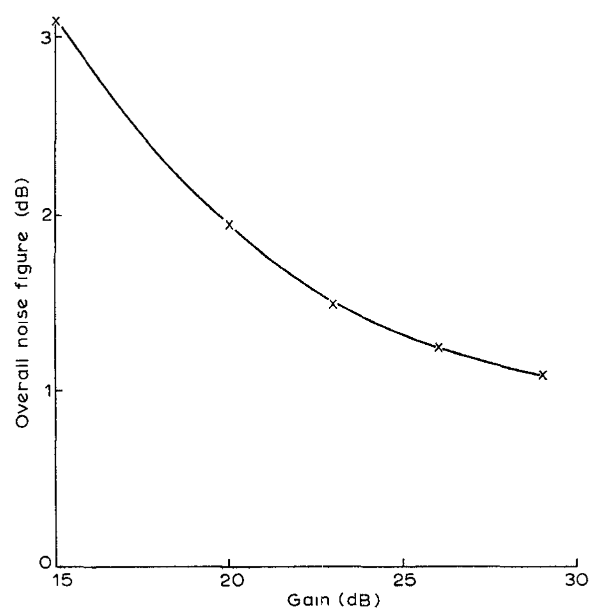


Fig. 8. Variation of overall noise figure with gain.

B. Amplifiers at S, C, and X bands

Similar amplifiers which have been constructed at *S*, *C*, and *X* bands are shown in Fig. 9. The design, construction, and testing was identical to that indicated in Section V-A. and the results together with the predicted performance are summarized in Table II.

In the case of the *X*-band amplifier, the diode series resonance frequency is itself at *X* band and Z_5 (Fig. 3) must be made low in order that the signal circuit will resonate in *X* band. In addition, the diode mounting was designed so as to reduce inductance with consequent increase in external stray capacity and reduction in parallel resonant frequency. Even with the low value of Z_5 required, the Q of the idler circuit is only increased by 15 percent.

VI. Improvement in Gain-Frequency Response by the Use of Filter Techniques

A. Theory

Theoretical considerations show that the gain-frequency response of a parametric amplifier may be improved by the use of bandpass filters for the signal and idling circuits rather than simple resonant circuits [5]–[7]. Humphreys [5] has calculated that the addition of one frequency-compensating circuit to both the signal and idling resonance of appropriate Q (to reduce the rate of change of reactance of both the signal and idling circuits) improves the gain frequency response since the gain-bandwidth product becomes $G^{1/4}B=k_2$ instead of $G^{1/2}B=k_1$ where k_1 and k_2 have a similar magnitude. A further resonant circuit added to the signal and idler response resonances should increase the gain frequency response further, since the relationship becomes of the form $G^{1/6}B=k_3$ where again k_3 is similar in magnitude to k_1 and k_2.

Figure 10 shows the circuit which Humphreys assumed for the analysis. C_0 is the diode capacitance and L its associated inductance. The network A contains lossless reactances arranged so that the combination of C_0, L, and A is resonant at both the signal (ω_1) and idling (ω_2) frequencies. The idling load is R_2 and the signal circuit is connected to a circulator. The first additional signal circuit C_1, L_1 is resonant at the signal frequency and C_1 is given by

$$C_1=\frac{1}{\omega_2{}^2C_0R_1R_2}$$

where R_1 is the circulator impedance. The first additional idling circuit C_2, L_2 is resonant at the idling frequency and C_2 is given by

$$C_2=\frac{1}{\omega_1{}^2C_0R_1R_2}.$$

Expressions can be similarly derived for C_1' and C_2'.

In practice it is more convenient to compensate the amplifier on the signal circuit alone and Appendix V shows that in this case the value of capacitance C_1 required for the first additional circuit is given by

$$C_1=\frac{1}{\pi R_1G^{1/2}B} \tag{5}$$

where R_1 is the circulator impedance and $G^{1/2}B$ is the observed gain-bandwidth product before the compensatory circuit has been added.

B. Practical Realization of Compensatory Network

It is often not convenient to form the required resonant circuit by means of lumped components and therefore a distributed circuit consisting of an open-circuited and short-circuited length of line is used. The rate of change of susceptance of the distributed circuit is arranged to be the same as for the lumped circuit. It is shown in Appendix VI that if l_1 is the distance from the short circuit and l_2 is the distance from the open circuit then the required characteristic admittance (Y_0) of the distributed parallel-resonant circuit is given by

$$Y_0=\frac{4C_1\omega_1}{\pi(2n+1)\operatorname{cosec}^2\beta l_1}$$

where C_1 is given by (5) and $l_1+l_2=(2n+1)\lambda/4$. Normally n is zero and Y_o is selected so that l_1 and l_2 are both approximately one-eighth of a wavelength and both the short circuit and open circuit are made adjustable.

It is physically more convenient to place this circuit a half wavelength away from the diode rather than at the diode and it is, therefore, placed across the junction of the 50-ohm line and the first section of the quarter-wave transformer. A second compensating circuit is placed a quarter of a wavelength away from the first and consists of a second parallel circuit since this is transformed to a series circuit in the plane of the first compensating circuit.

The compensating circuits are adjusted to obtain the required gain-frequency response with the aid of a swept signal source and an oscilloscope.

C. Results

Figure 11 shows an *S*-band amplifier with and without the additional compensating circuits. Figure 12 shows the gain-frequency responses taken from the oscilloscope. It can be seen that the uncompensated 3 dB bandwidth is 54 MHz at 20 dB gain and becomes 200 MHz with one additional circuit and 280 MHz with two additional circuits. Thus the bandwidth has been increased by factors of 3.7 and 5.6.

The noise performance of the amplifier has been investigated by measurement of the noise figure as the local oscillator frequency was varied so that the noise figure of the system was determined at various frequencies within the bandpass characteristic of the amplifier. Figure 13 shows this for the uncompensated amplifier and for the compensated amplifier. From the graphs the variation of amplifier noise figure over the bandpass characteristic can be derived. It can be seen that the compensatory circuits increase the noise figure by approximately 0.3 dB. This increase in noise figure arises from losses associated with the compensatory elements and could be reduced by a better design of variable short circuits.

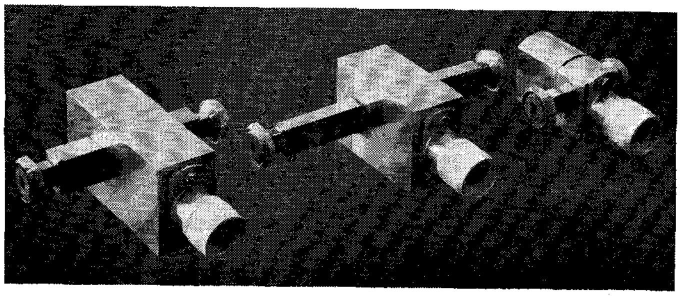


Fig. 9. S, C, and X band amplifiers.

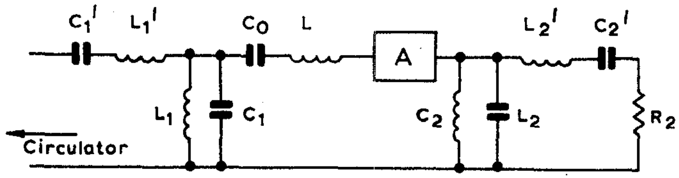


Fig. 10. Circuit of parametric amplifier with compensatory circuits at signal and idling frequencies.

TABLE II

Diode	Operating Frequency (HGz)	Current (μA)	R_g/R_d	Idler Frequency (GHz)	3 dB Bandwidth at 20 dB Gain (MHz)		Noise Temperature	
					Predicted	Measured	Predicted	Measured
VX3368	3.0	1	6.0	30	42	40	76°K	95°K
VX3368	5.6	1	3.8	32	100	100	142°K	139°K
VX3368 Selected	8.2	4	2.8	21.5	100	40	226°K	270°K

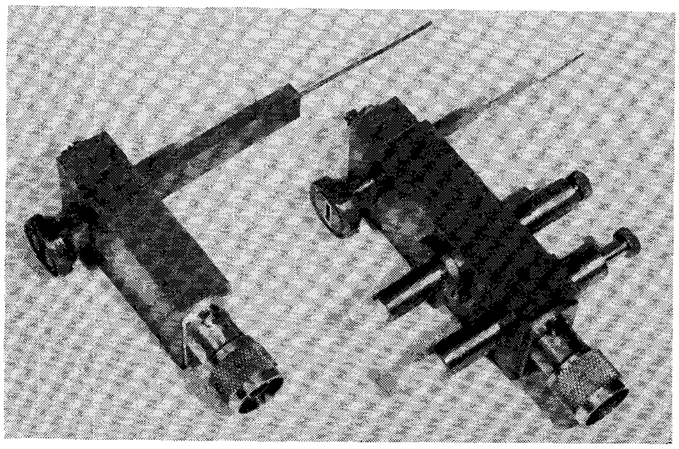


Fig. 11. The second amplifier has two additional compensating circuits added to the signal circuit.

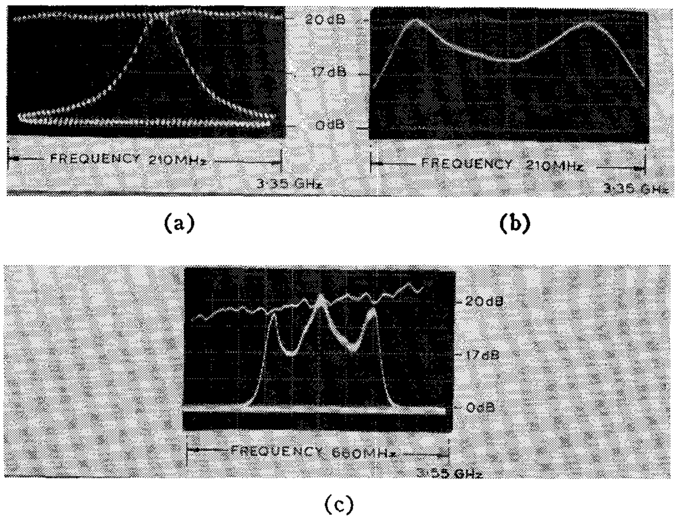


Fig. 12. Gain frequency responses obtained with and without compensatory circuits. (a) Simple circuit 3 dB bandwidth 54 MHz. (b) Simple circuit, plus one compensating circuit, 3 dB bandwidth 200 MHz. (c) Simple circuit, plus two compensating circuits, 3 dB bandwidth 280 MHz.

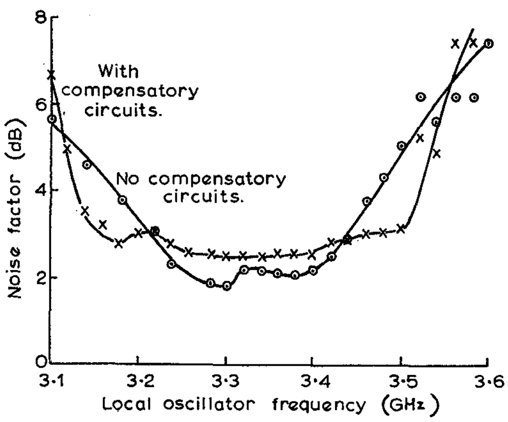


Fig. 13. Noise factor of amplifier with and without two compensatory circuits.

VII. Conclusions

A simple parametric amplifier which can readily be designed for operation at S, C, and X band has been described and it has been shown that the results are largely in agreement with the theoretical predictions.

Appendix I

The analysis below derives an expression for the condition that three sections of coaxial line which are a quarter-wavelength at the idling frequency shall be equivalent at the signal frequency to a single quarter-wavelength of a specified impedance.

Figure 14 shows three cascaded sections of lossless line with impedances Z_1, Z_2, and Z_1 and lengths l_1, l_2, and l_1 which can be represented by the transfer matrices M_1 M_2 and M_1, respectively, where

$$M_1 = \begin{vmatrix} A_1 & B_1 \\ C_1 & D_1 \end{vmatrix}, \qquad M_2 = \begin{vmatrix} A_2 & B_2 \\ C_2 & D_2 \end{vmatrix}$$

in which

$$A_1 = D_1 = \cos\beta l_1, \qquad A_2 = D_2 = \cos\beta l_2$$

$$B_1 = jZ_1 \sin\beta l_1, \qquad B_2 = jZ_2 \sin\beta l_2,$$

$$C_1 = \frac{j}{Z_1}\sin\beta l_1, \qquad C_2 = \frac{j}{Z_2}\sin\beta l_2.$$

The matrix for the cascaded system (M_T) is given by

$$M_T = \begin{vmatrix} A_T & B_T \\ C_T & D_T \end{vmatrix}. \tag{6}$$

The matrix of a quarter-wavelength of line of impedance Z_e is given by

$$\begin{vmatrix} 0 & jZ_e \\ jY_e & 0 \end{vmatrix}. \tag{7}$$

Equating (6) and (7)

$$A_T = 0, \tag{8}$$

$$B_T = jZ_e \tag{9}$$

and

$$C_T = jY_e.$$

Equation (8) gives

$$\frac{Z_1}{Z_e} = \frac{1}{\tan 2\beta l_1 \tan\beta l_2} \pm \sqrt{\frac{1}{(\tan 2\beta l_1 \tan\beta l_2)^2} - 1}. \tag{10}$$

Equation (9) gives

$$\frac{Z_1}{Z_2} = \sin 2\beta l_1 \cos\beta l_2 + \sin\beta l_2 \frac{Z_1}{Z_2}\cos^2\beta l_1 - \frac{Z_2}{Z_1}\sin^2\beta l_1. \tag{11}$$

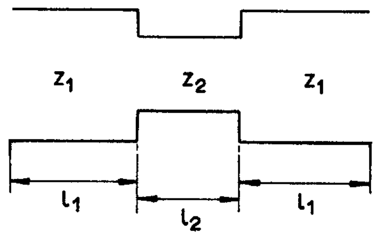


Fig. 14. The three cascaded sections of line of lengths l_1, l_2, and l_1 and impedances Z_1, Z_2, and Z_1.

If we now arrange at some other frequency f_2 where $f_2 > f_1$, that $l_1 = l_2 = \lambda_2/4$ then

$$\frac{Z_1}{Z_2} = \frac{1}{\tan \pi \frac{f_1}{f_2} \tan \frac{\pi}{2}\frac{f_1}{f_2}} \pm \sqrt{\frac{1}{\left(\tan \pi \frac{f_1}{f_2} \tan \frac{\pi}{2}\frac{f_1}{f_2}\right)^2} - 1} \tag{12}$$

and

$$\frac{Z_1}{Z_e} = \sin \pi \frac{f_1}{f_2} \cos \frac{\pi}{2}\frac{f_1}{f_2} + \sin \frac{\pi}{2}\frac{f_1}{f_2}\left(\frac{Z_1}{Z_2}\cos^2\frac{\pi}{2}\frac{f_1}{f_2} - \frac{Z_2}{Z_1}\sin^2\frac{\pi}{2}\frac{f_1}{f_2}\right). \tag{13}$$

In order that Z_1/Z_2 shall be real and positive it can be shown from (10) that it is necessary that $f_2 > 3f_1$.

Equations (11) and (12) are used to calculate Z_1 and Z_2 since f_1 and f_2 are known and Z_e is specified.

The input impedance at f_2 is given by $Z_1^4/Z_2^2 Y_L$ where Y_L is the load admittance and this expression is small provided $Z_1 \ll Z_2$.

Appendix II

The analysis below derives an expression for the Q of the idling circuit including the effect of the distributed quarter-wavelength short-circuited line.

The circuit to which the idling energy is confined is shown in Fig. 15, in which the equivalent circuit of the diode consisting of junction capacitance C_j,[1] spreading resistance R_S,[1] lead inductance L, and stray capacitance C_2 is terminated in a short-circuited line of impedance Z_0. The introduction of idling energy into this circuit is represented by the voltage generator e.

In order to calculate the Q of this circuit it is convenient first to transform it to an equivalent series circuit. Representing the susceptance of C_2 by B_1 and the susceptance of the line, Z_0, by B_2, the equivalent reactance X_1 is given by $(B_1+B_2)^{-1}$. The rate of change of this reactance with fre-

[1] The internal stray C_1, of Fig. 1, of the diode is incorporated into the effective junction capacity C_j and effective spreading resistance R_s.

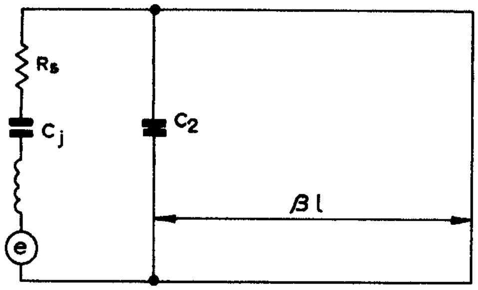


Fig. 15. Idling energy circuit.

quency is given by

$$\frac{\delta X_1}{\delta\omega} = \frac{\frac{\delta B_1}{\delta\omega} + \frac{\delta B_2}{\delta\omega}}{(B_1 + B_2)^2} \cdot$$

Q is defined for a series circuit by

$$Q = \frac{\omega_0}{2R}\,\frac{\delta X}{\delta\omega}$$

so that the Q for the resultant series circuit is given by

$$Q = \frac{\omega_0}{2R_s}\left\{\frac{C_2 + \frac{1}{Z_0 c}\operatorname{cosec}^2\frac{\omega_0 l}{c}}{\left(\omega C_2 - Y_0\cos\frac{\omega l}{c}\right)^2} + \frac{1}{\omega_0^2 C_j} + L\right\}$$

where c is the velocity of light.

We can now substitute the condition $l=\lambda/4$ and $\omega_0^2=1/LC'$ where $1/C'=1/C_j+1/C_2$ corresponding to resonance of the lumped components and resonance of the short-circuited line. This gives

$$Q = \frac{\omega_0 L}{R_s}\left[1 + \frac{\lambda}{8C_s^2 Z_0 c\omega_0^2 L}\right].$$

But the Q of the lumped components alone, Q_1, is given by

$$Q_1 = \frac{\omega_0 L}{R_s},$$

so that

$$\frac{Q}{Q_1} = 1 + \frac{1}{32\pi^2 C_s^2 Z_0 L f^3} \cdot$$

Appendix III

The analysis derives the γf_c product for the diode in terms of the signal frequency and overcoupling ratio.

The usual series parametric amplifier which is analyzed is shown in Fig. 16. The capacitance C_0 is resonant with L_1 at the signal frequency ω_1, and L_2 at the idling frequency ω_2. R_d is the internal loss in the signal circuit, R_g is the generator resistance at the signal frequency, r_2 is the loss at the idling frequency. Filters F_1 and F_2 are provided to restrict the signal and idling currents to the appropriate circuits.

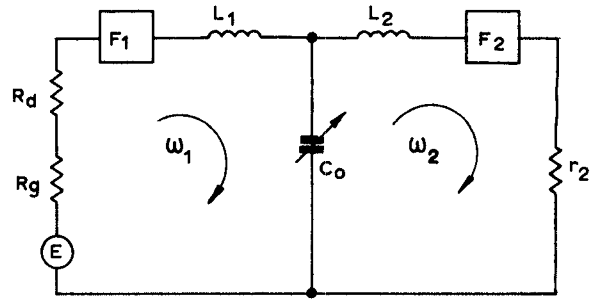


Fig. 16. Parametric amplifier circuit.

Conventional analysis shows that, with the signal and idling circuits resonant, the ratio of negative resistance to positive resistance A is given by

$$A = \frac{\gamma^2}{C_0^2\omega_1\omega_2 r_1 r_2} \tag{14}$$

where

$$r_1 = R_d + R_g$$

and

$$\gamma = \frac{C_{\max} - C_{\min}}{2(C_{\max} + C_{\min})} \cdot$$

$C_{\max}$ and $C_{\min}$ are the maximum and minimum values of the diode capacitance produced by the pump. Equation (14) can be written as

$$\frac{\omega_1\omega_2}{\omega_c^2}\left(1 + \frac{R_g}{R_d}\right) = \gamma^2$$

where

$$\omega_c = \frac{1}{C_0 R_d}, \quad A \text{ is unity and } \gamma^2 = R_D$$

so that

$$\gamma f_c = \sqrt{f_1 f_2\left(1 + \frac{R_g}{R_d}\right)}. \tag{15}$$

Equation (15) expresses the diode figure of merit γf_c in terms of signal frequency, idling frequency, and overcoupling ratio at high gain.

Appendix IV

In a negative resistance parametric amplifier it is not convenient to measure the unpumped idling circuit bandwidth by the conventional technique of coupling weakly to the resonant structure and it is therefore of interest to derive an expression for the idling bandwidth from the observed variation of signal frequency as the pump frequency is varied.

Consider an amplifier which operates at the mid-band signal frequency ω_1, when pumped at ω_3 corresponding to the mid-band idling frequency ω_2. If ω_3 is increased to ω_p so that $\omega_p=\omega_3(1+\delta_3)$ then we can also write

$$\omega_s = \omega_1(1 + \delta_1) \quad \text{and} \quad \omega_i = \omega_2(1 + \delta_2).$$

But $\omega_s + \omega_i = \omega_p$ so that

$$\delta_2 = \frac{\delta_3\omega_3 - \delta_1\omega_1}{\omega_2} \cdot \tag{16}$$

We know that the current in the signal circuit i can be written as

$$i = \frac{e/r_1}{1 + j2\delta_1 Q_1 - \dfrac{A}{1 - 2j\delta_2 Q_2}}$$

where e is the signal circuit EMF and r_1 the total signal circuit resistance. The modulus of the denominator of this expression $|D|$ can be written as

$$|D|^2 = (1 - A)^2 + 4(\delta_1 Q_1 - \delta_2 Q_2)^2. \tag{17}$$

Differentiation of (17) and subsequent manipulation yields

$$\frac{\delta_1}{\delta_3} = \frac{\dfrac{\omega_3}{\omega_2}}{\dfrac{Q_1}{Q_2} + \dfrac{\omega_1}{\omega_2}} \cdot \tag{18}$$

We now wish to express this in terms of the slope of the measured pump frequency-to-signal frequency curve. Writing $\Delta\omega_1$ as $\omega_1(1+\delta_1) - \omega_1$ and $\Delta\omega_3$ as $\omega_3(1+\delta_3) - \omega_3$ (7) can be written as

$$B_i = B_s\left(\frac{\Delta\omega_3}{\Delta\omega_1} - 1\right) \tag{19}$$

where B_i is the unpumped bandwidth of the idling circuit and B_s is the unpumped bandwidth of the signal circuit.

Appendix V

It is convenient when compensating a parametric amplifier at the signal circuit also to express the required capacitance of the first compensatory circuit in terms of the gain-bandwidth product of the uncompensated amplifier. The analysis below derives the expression.

It can be shown that the impedance of the signal circuit of a pumped negative resistance amplifier can be written in the form

$$r_1\left\{(1 - A) - j\left(A\frac{\delta X_2}{r_2} - \frac{\delta X_1}{r_1}\right)\right\} \tag{20}$$

in which

r_1 = the total loss in the signal circuit,
r_2 = the total loss in the idling circuit,
δX_1 = the reactance of the umpumped signal circuit which is assumed close to resonance,
δX_2 = the reactance of the unpumped idling circuit which is assumed close to resonance, and
A = the ratio of negative resistance to positive resistance in the signal circuit.

It can be seen that the real part of the impedance is independent of small changes in operating frequency and the imaginary part is frequency dependent. The signal frequency equivalent circuit consists, therefore, of a series combination of positive resistance, negative resistance, and series resonant circuit of appropriate Q.

Now we can derive the operating bandwidth of the amplifier B_{op} from (20) and it is given by

$$B_{op} = \frac{(1 - A)}{\dfrac{1}{B_1} + \dfrac{1}{B_2}} = B_{eff}(1 - A). \tag{21}$$

where B_1 is the unpumped signal circuit bandwidth, and B_2 is the unpumped idling circuit bandwidth, and B_{eff} is the effective bandwidth of the signal circuit.

Figure 17 shows a series resonant circuit consisting of a capacitance C, inductance L, and resistance r which is connected to an external resistance R. The circuit is resonant at the frequency ω. Across R is connected a parallel resonant system of inductance L' and capacitance C'. C' and L' are also resonant at ω.

Analysis of the equivalent series circuit shows that there is reactance compensation for the condition.

$$C'R = \frac{1}{\omega^2 CR} \cdot \tag{22}$$

This condition is independent of r.

Now the bandwidth (B_0) of the uncompensated circuit is given by

$$B_0 = \frac{\omega^2 CR}{2\pi} \quad \text{when } R \gg r$$

so (22) can be written as

$$C'R = \frac{1}{2\pi B_0} \cdot \tag{23}$$

Substitution of (23) into (22) gives

$$C'R = \frac{1}{2\pi}\left(\frac{1}{B_1} + \frac{1}{B_2}\right).$$

But we know that

$$G^{1/2}B_0 = 2\left(\frac{1}{B_1} + \frac{1}{B_2}\right)^{-1}$$

so that

$$C'R = \frac{1}{\pi G^{1/2} B_0} \tag{24}$$

which expresses C' in terms of the uncompensated circuit $G^{1/2}B_0$ product and the external loading on the signal circuit, R.

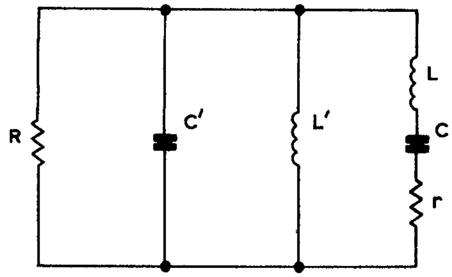


Fig. 17. Series circuit with shunt compensatory network.

Appendix VI

When it is not physically convenient to provide a compensatory shunt resonant circuit using lumped components, it is necessary to use distributed components to achieve the same rate of change of reactance as with the lumped-shunt circuit.

Figure 18 shows a short-circuited line of length l_1 in shunt with an open-circuited line of length l_2. The characteristic admittance is Y_0.

The total susceptance B of this system across AA' is given by

$$B = -Y_0 \cot \beta l_1 + Y_0 \tan \beta l_2$$

and

$$\frac{\delta B}{\delta \omega} = \frac{Y_0}{c}(l_1 + l_1 \cot^2 \beta l_1 + l_2 + l_2 \tan^2 \beta l_2)$$

where c is the velocity of light.

Since the system is resonant we can write

$$l_1 + l_2 = \frac{(2n+1)\lambda}{4}$$

and $\delta B/\delta \omega$ becomes

$$\frac{Y_0}{c}(2n+1)\frac{\lambda}{4}(\operatorname{cosec}^2 \beta l_1). \tag{25}$$

Now the lumped shunt resonant circuit with capacitance C has a rate of change of susceptance $\delta\beta/\delta\omega$ given by

$$\frac{\delta B}{\delta \omega} = 2C. \tag{26}$$

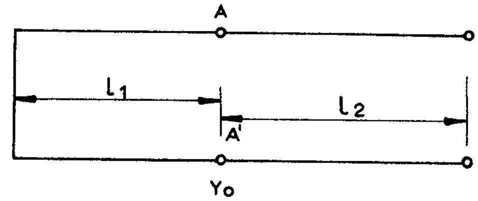


Fig. 18. Short-circuited line l_1 in shunt with open-circuited line l_2 with characteristic admittance Y_0.

Equating (25) and (26) we get

$$Y_0 = \frac{4C\omega}{\pi(2n+1)\operatorname{cosec}^2 \beta l_1}$$

which expresses the characteristic impedance Y_0 and length l_1 for the distributed system to have a rate of change of susceptance with frequency equal to that of a lumped capacitance C and a lumped inductance with the same resonant frequency as the distributed circuit.

Acknowledgment

Acknowledgment is made to D. A. E. Roberts, B. J. Rickett, and A. P. Tod for valuable discussions and some of the measurements reported in the paper.

References

[1] C. S. Aitchison, B. L. Hymphreys, and E. L. Neufeld, "The overall noise figures of diode parametric amplifier systems," *Proc. IEE (London)*, vol. 110, pp. 348–352, February 1963.
[2] D. A. E. Roberts, "Measurement of varactor diode impedance," *IEEE Trans. on Microwave Theory and Techniques (Correspondence)*, vol. MTT-12, pp. 471–475, July 1964.
[3] D. A. E. Roberts and K. Wilson, "Evaluation of high quality varactor diodes," *Proc. of the 1965 Symp. on Microwave Application of Semiconductors.*
[4] M. Uenohara and J. P. Elward, Jr., "Parametric amplifiers for high sensitivity receivers," *IEEE Trans. on Antennas and Propagation*, vol. AP-12, p. 940, December 1964.
[5] B. L. Humphreys, "Characteristics of broadband parametric amplifiers using filter networks," *Proc. IEE (London)*, vol. 111, p. 264–274, February 1964.
[6] G. L. Matthaei, "A study of the optimum design in wideband parametric amplifiers and upconverters," *IRE Trans. on Microwave Theory and Techniques*, vol. MTT-9, pp. 23–38, January 1961.
[7] J. T. De Jaeger, "Maximum bandwidth performance of a nondegenerate parametric amplifier with single tuned idler circuit," *IEEE Trans. on Microwave Theory and Techniques*, vol. MTT-12, pp. 459–467, July 1964.